

MATHEMATICS

Chapter 9: Area of Parallelograms and Triangles



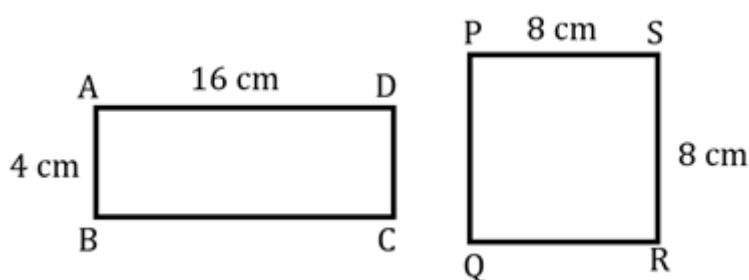
Area of Parallelograms and Triangles

Introduction to Planar region and Area

The part of the plane enclosed by a simple closed figure is called a **planar region** corresponding to that figure. The magnitude or measure of that planar region is called its **area**.

Congruent figures and their areas

- Two figures are called **congruent**, if they have the same shape and the same size.
- If two figures A and B are **congruent**, they must have **equal areas**.
- Two figures having **equal areas need not be congruent**. In the figure,



Area of rectangle ABCD = $16 \times 4 = 64 \text{ cm}^2$

Area of square PQRS = $8^2 = 64 \text{ cm}^2$

Area of rectangle ABCD = Area of square PQRS

But rectangle ABCD and square PQRS are not congruent.

Area of a figure

Area of a figure is a number (in some unit) associated with the part of the plane enclosed by the figure.

Area of the planar region

If a planar region formed by a figure T is made up of two non-overlapping planar regions formed by figures P and Q, then $\text{ar}(T) = \text{ar}(P) + \text{ar}(Q)$.

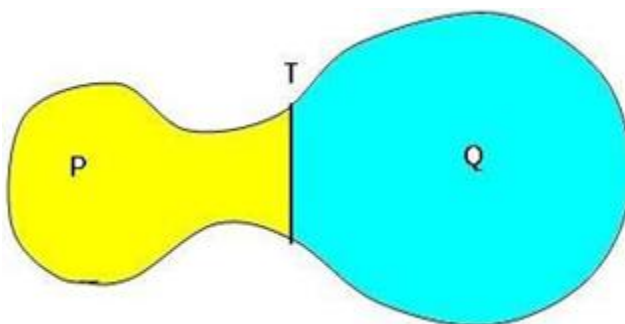
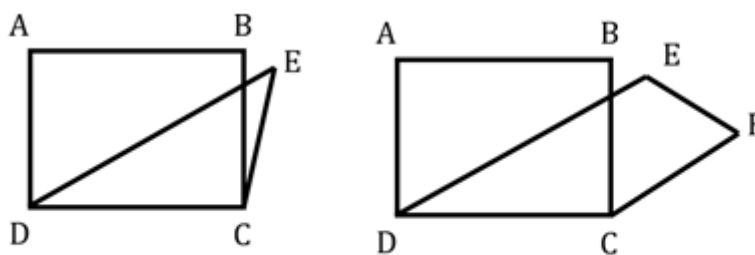


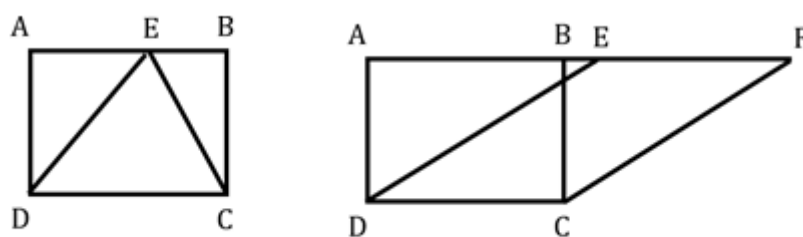
Figure on the same base and between the same parallels

- Two figures are said to be on the same base and between the same parallels if they

have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.



On the same base but not between the same parallels

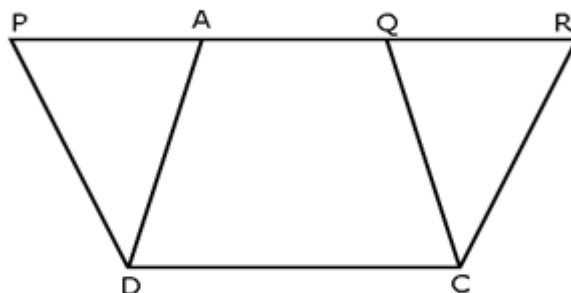


On the same base CD and between the same parallels AF and CD

- Please note that out of the two parallels, one must be the line containing the common base.

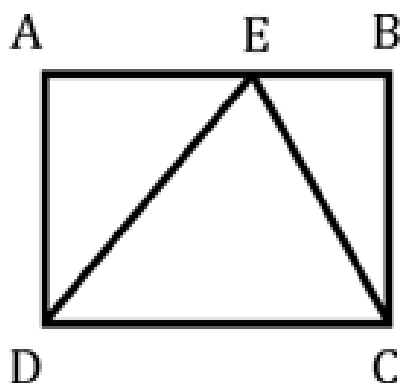
Areas of figures on the same base and between the same parallels

- Parallelograms on the same base and between the same parallels are equal in area.



In the figure, parallelograms PQCD and ARCD lie on the same base CD and between same parallels CD and PR. So, $\text{ar}(\text{PQCD}) = \text{ar}(\text{ARCD})$.

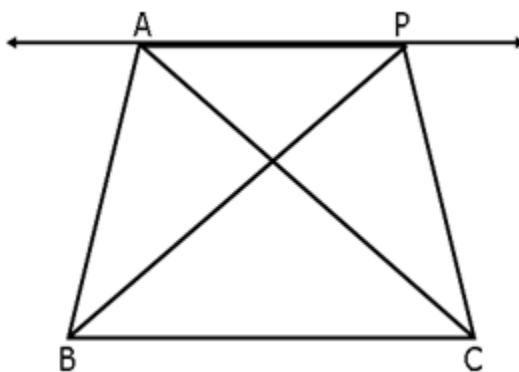
- **Area of a parallelogram** is the product of its any side and the corresponding altitude.
- Parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels.
- If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half of the area of the parallelogram.



In the figure, triangle DEC and parallelogram ABCD are on the same CD and between the same parallels AB and CD.

Therefore, area of triangle DEC = $\frac{1}{2} \times$ area of parallelogram ABCD.

- Two triangles on the same base (or equal base) and between the same parallel are equal in area.



In the figure, triangles ABC and PBC lie on the same base BC and between same parallels BC and AP.

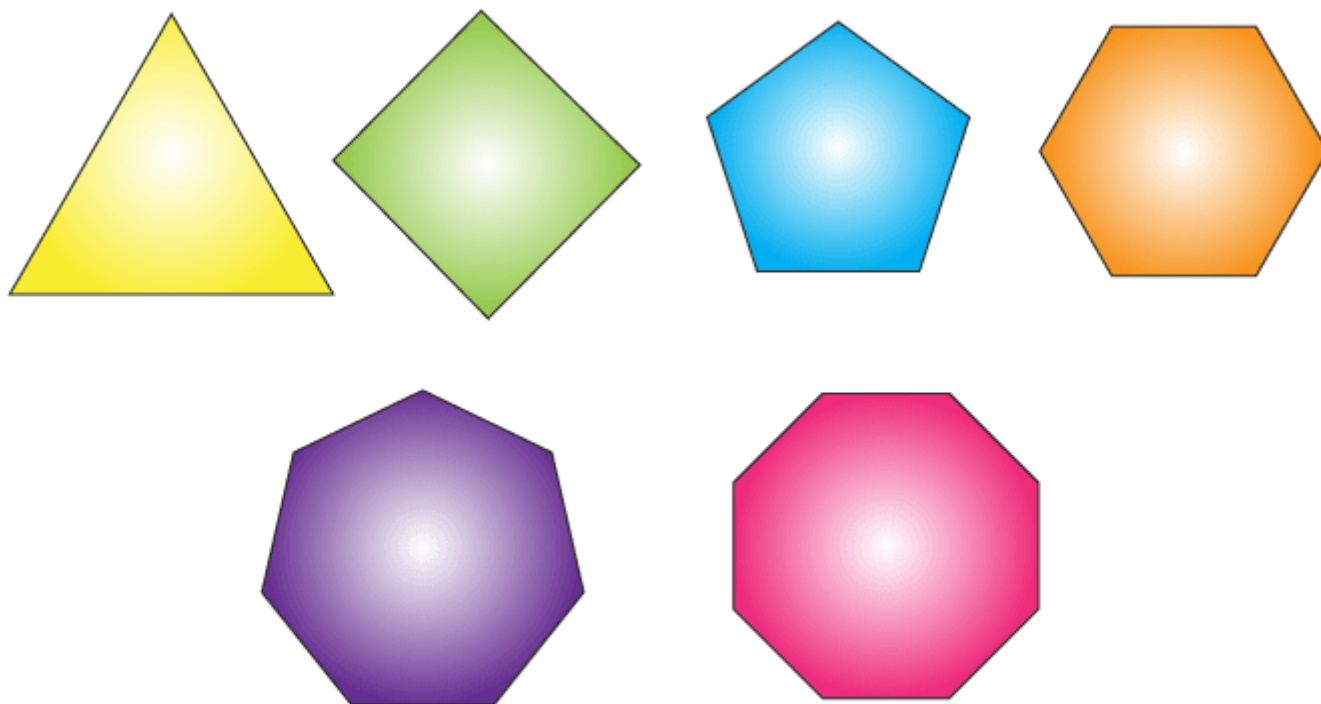
Therefore, $\text{ar}(\text{triangle ABC}) = \text{ar}(\text{triangle PBC})$.

- Area of a triangle** is half the product of its base (or any side) and the corresponding altitude (or height).

Important facts about triangles on the same base

- Two triangles with same base (or equal bases) and equal areas will have equal corresponding altitudes.
- Two triangles having the same base (or equal bases) and equal areas lie between the same parallels.
- A median of a triangle divides it into triangles of equal areas.

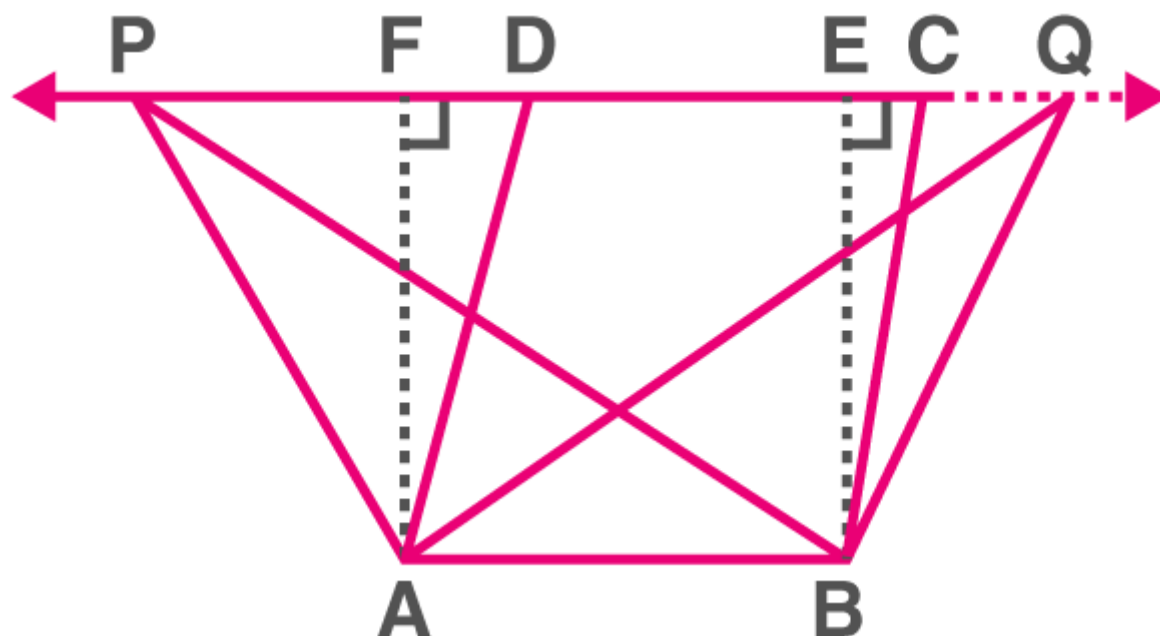
The area represents the amount of planar surface being covered by a closed geometric figure.



Figures on the Common Base and Between the Same Parallels

Two shapes are stated to be on the common base and between the same parallels if:

- They have a common side.
- The sides parallel to the common base and vertices opposite the common side lie on the same straight line parallel to the base.

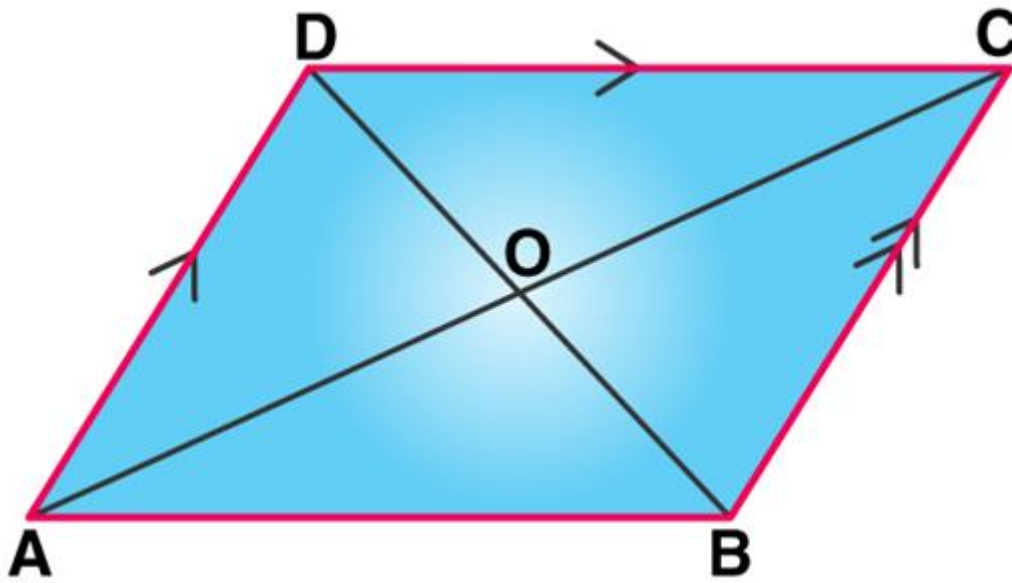


For example Parallelogram ABCD, Rectangle ABEF and Triangles ABP and ABQ

Area of a parallelogram

The area of a parallelogram is the region bounded by the parallelogram in a given two-dimension space. To recall, a parallelogram is a special type of quadrilateral which has four sides and the pair of opposite sides are parallel. In a parallelogram, the opposite sides are

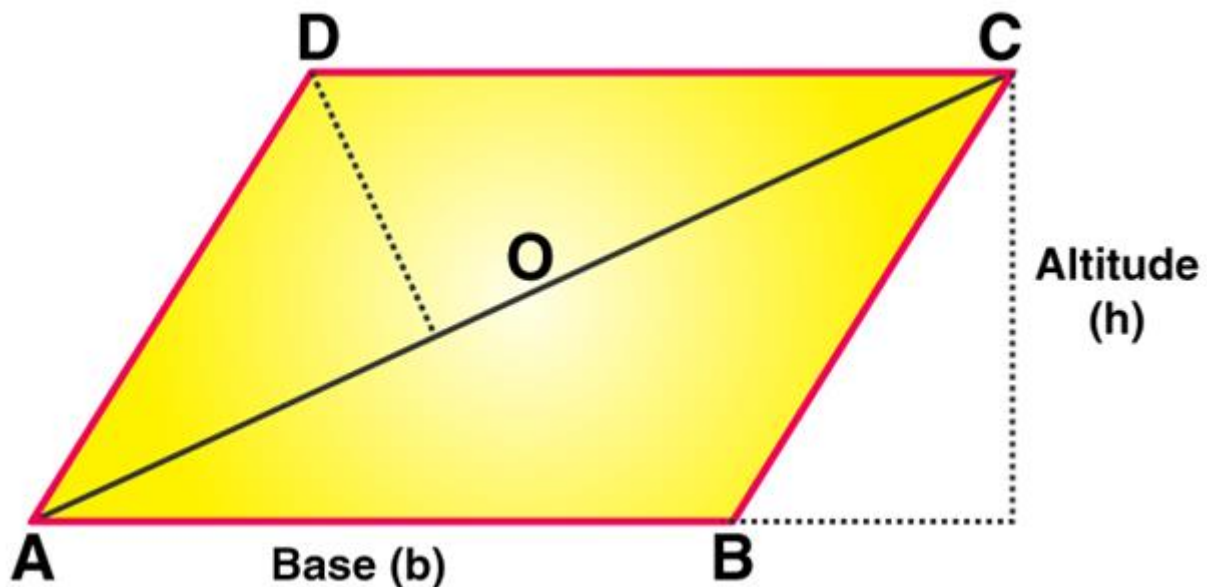
of equal length and opposite angles are of equal measures. Since the rectangle and the parallelogram have similar properties, the area of the rectangle is equal to the area of a parallelogram.



Parallelogram

Area of Parallelogram Formula

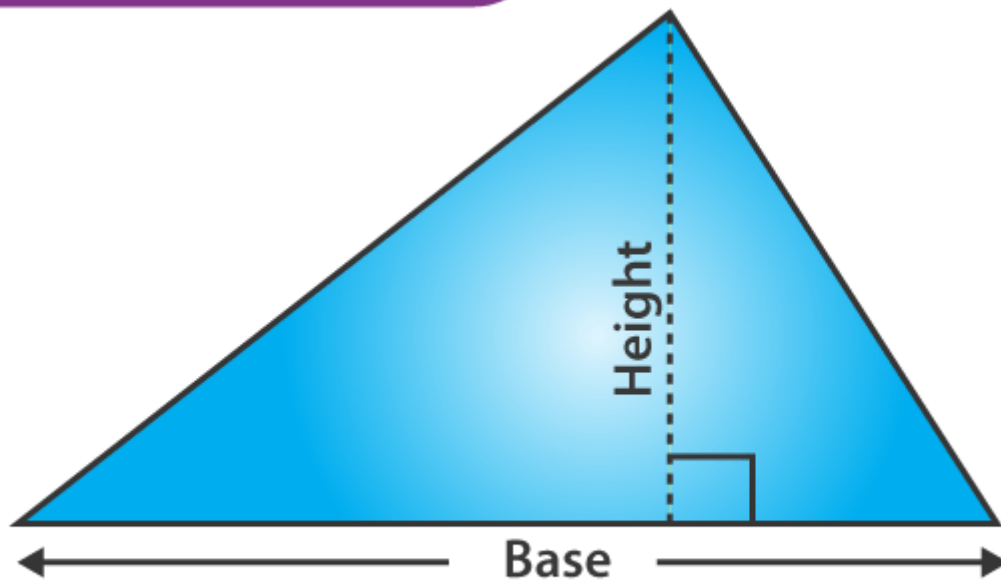
To find the area of the parallelogram, multiply the base of the perpendicular by its height. It should be noted that the base and the height of the parallelogram are perpendicular to each other, whereas the lateral side of the parallelogram is not perpendicular to the base. Thus, a dotted line is drawn to represent the height.



Area of a parallelogram = $b \times h$

Where 'b' is the base and 'h' is the corresponding altitude (Height).

Area of a triangle

AREA OF TRIANGLE

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

Area of a Triangle Formula

The area of the triangle is given by the formula mentioned below:

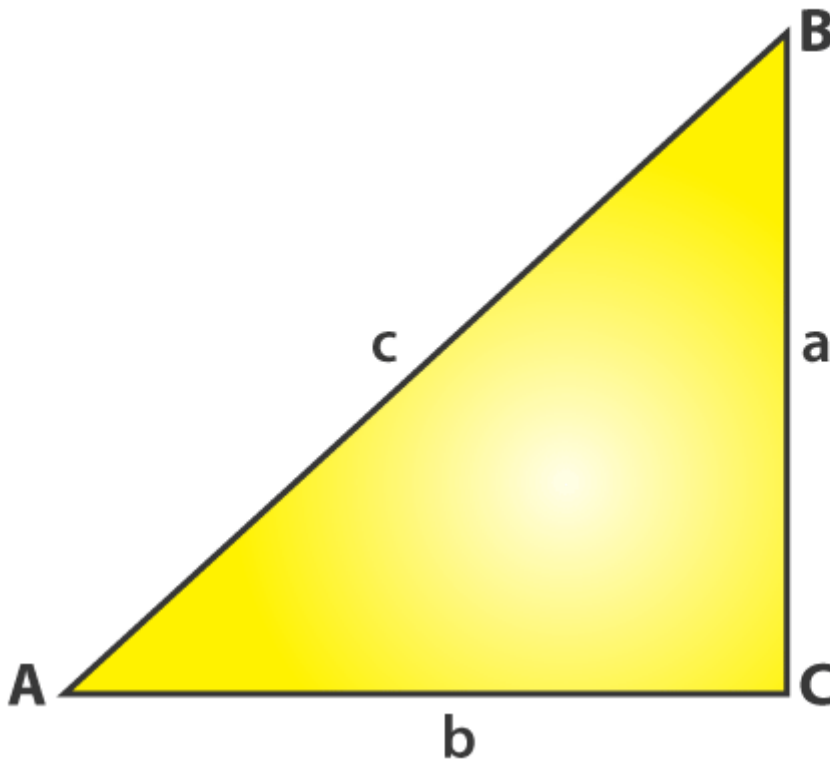
Area of a Triangle = $A = \frac{1}{2} (b \times h)$ square units

where b and h are the base and height of the triangle, respectively.

Now, let's see how to calculate the area of a triangle using the given formula. The area formulas for all the different types of triangles like an area of an equilateral triangle, right-angled triangle, an isosceles triangle are given below. Also, how to find the area of a triangle with 3 sides using Heron's formula with examples.

Area of a Right-Angled Triangle

A right-angled triangle, also called a right triangle has one angle at 90° and the other two acute angles sums to 90° . Therefore, the height of the triangle will be the length of the perpendicular side.

AREA OF A RIGHT ANGLED TRIANGLE

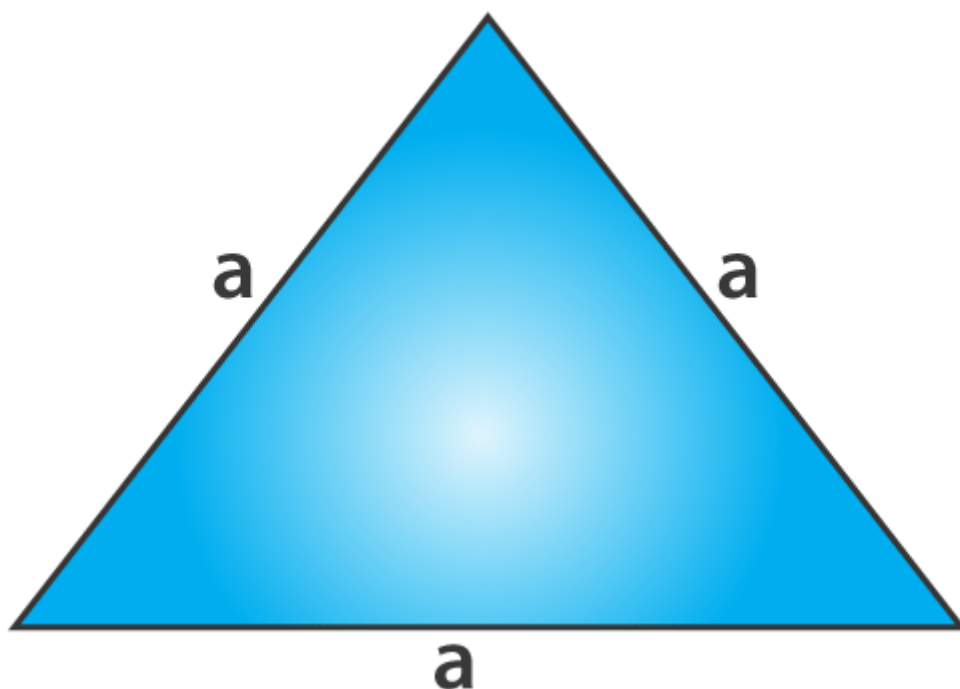
Area of a Right Triangle = $A = \frac{1}{2} \times \text{Base} \times \text{Height (Perpendicular distance)}$

From the above figure,

Area of triangle ACB = $\frac{1}{2} ab$

Area of an Equilateral Triangle

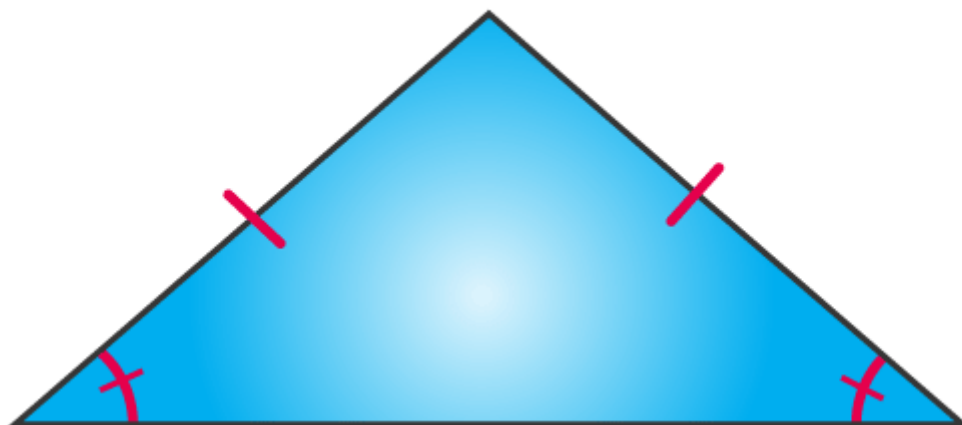
An equilateral triangle is a triangle where all the sides are equal. The perpendicular drawn from the vertex of the triangle to the base divides the base into two equal parts. To calculate the area of the equilateral triangle, we have to know the measurement of its sides.

AREA OF AN EQUILATERAL TRIANGLE

Area of an Equilateral Triangle = $A = (\sqrt{3})/4 \times \text{side}^2$

Area of an Isosceles Triangle

An isosceles triangle has two of its sides equal and also the angles opposite the equal sides are equal.

AREA OF AN ISOSCELES TRIANGLE

Area of an Isosceles Triangle = $\frac{1}{4} b\sqrt{4a^2 - b^2}$

Perimeter of a Triangle

The perimeter of a triangle is the distance covered around the triangle and is calculated by adding all three sides of a triangle.

The perimeter of a triangle = $P = (a + b + c)$ units

where a, b and c are the sides of the triangle.

Area of Triangle with Three Sides (Heron's Formula)

The area of a triangle with 3 sides of different measures can be found using Heron's formula. Heron's formula includes two important steps. The first step is to find the semi perimeter of a triangle by adding all the three sides of a triangle and dividing it by 2. The next step is that, apply the semi-perimeter of triangle value in the main formula called "Heron's Formula" to find the area of a triangle.

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

where, s is semi-perimeter of the triangle $= s = (a + b + c) / 2$

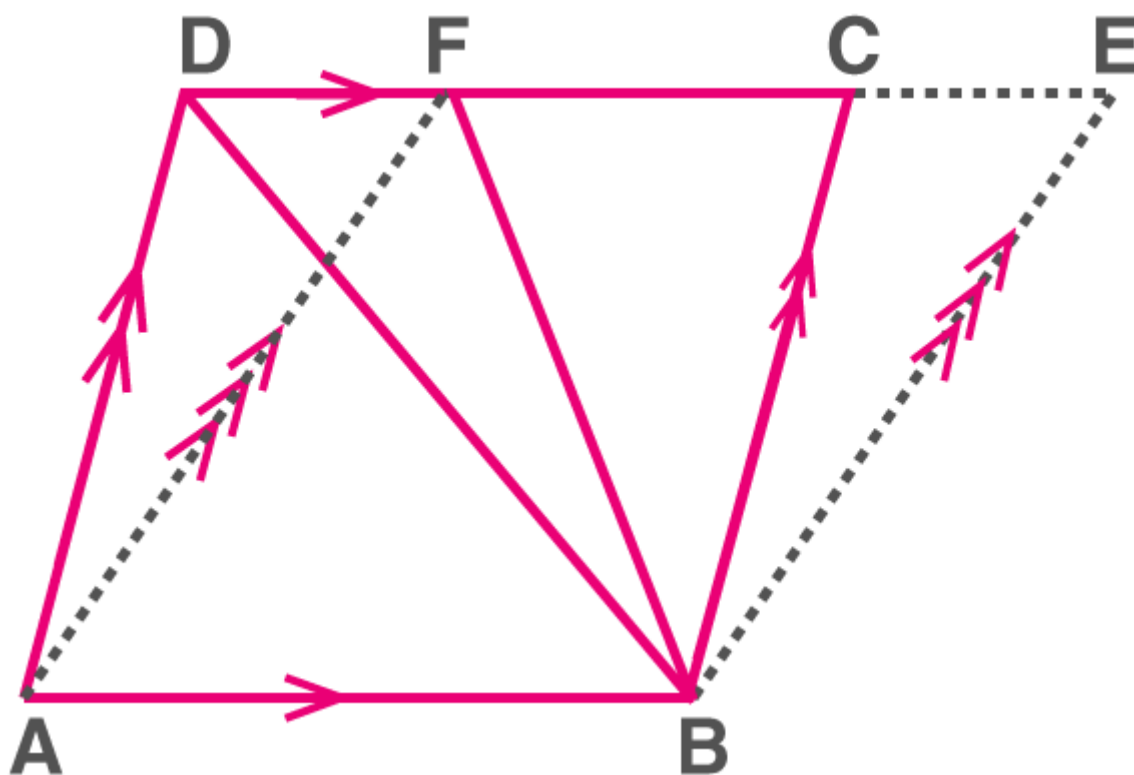
We have seen that the area of special triangles could be obtained using the triangle formula. However, for a triangle with the sides being given, the calculation of height would not be simple. For the same reason, we rely on Heron's Formula to calculate the area of the triangles with unequal lengths.

Theorems

Parallelograms on the Common Base and Between the Same Parallels

Two parallelograms are said to be on the common/same base and between the same parallels if

- They have a common side.
- The sides parallel to the common side lie on the same straight line.



Parallelogram ABCD and ABEF

Theorem: Parallelograms that lie on the common base and between the same parallels are said to have equal in area.

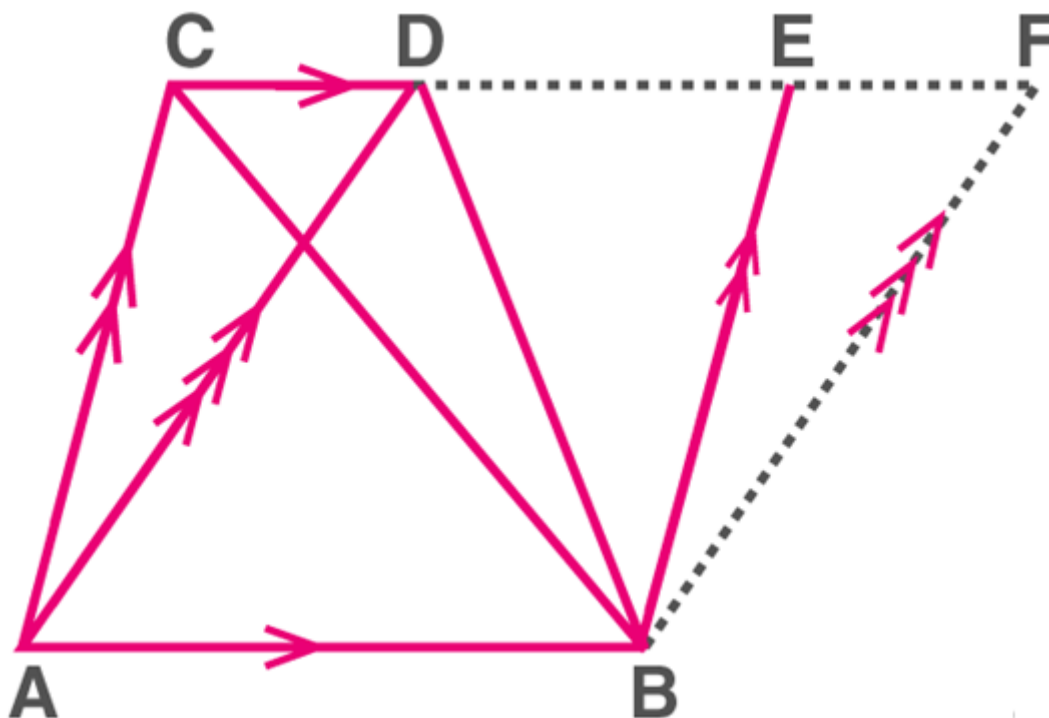
Here, $\text{ar}(\text{parallelogram } ABCD) = \text{ar}(\text{parallelogram } ABEF)$

Triangles on the Common Base and Between the Same Parallels

Two triangles are said to be on the common base and between the same parallels if

They have a common side.

The vertices opposite the common side lie on a straight line parallel to the common side.

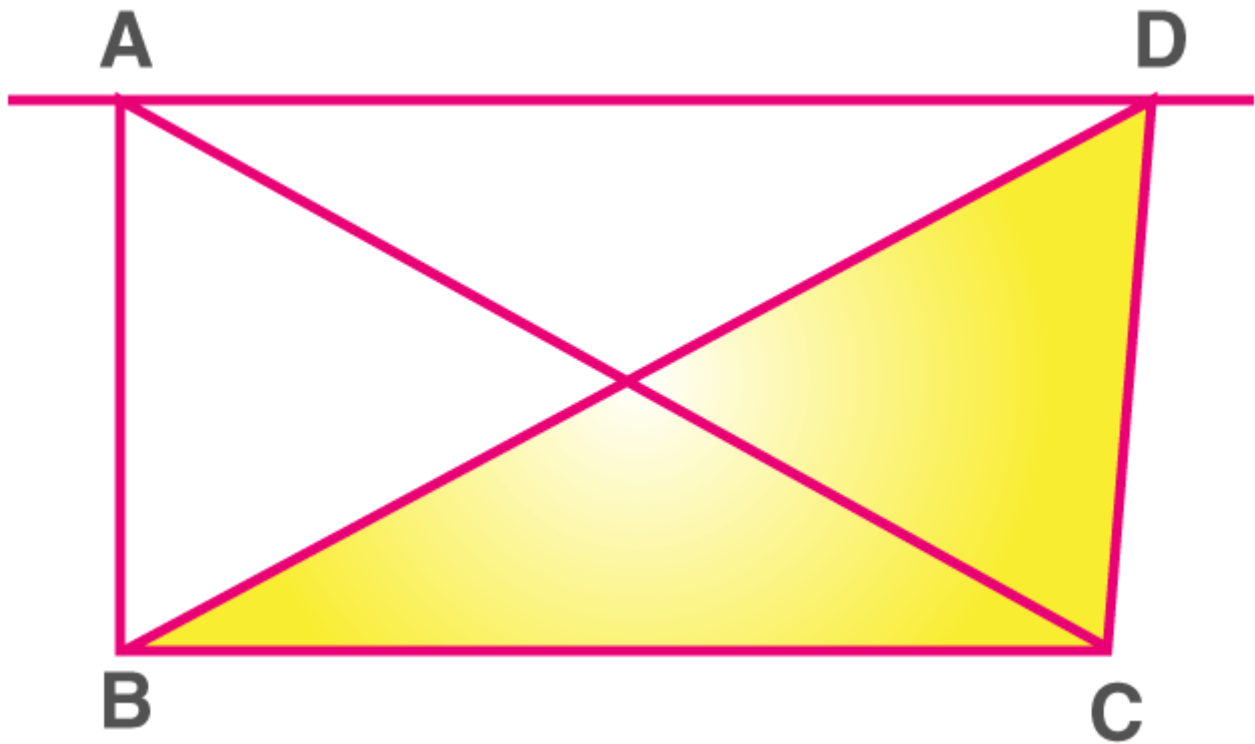


Triangles ABC and ABD

Theorem: Triangles that lie on the same or the common base and also between the same parallels are said to have an equal area.

Here, $\text{ar}(\triangle ABC) = \text{ar}(\triangle ABD)$

Two Triangles Having the Common Base & Equal Areas

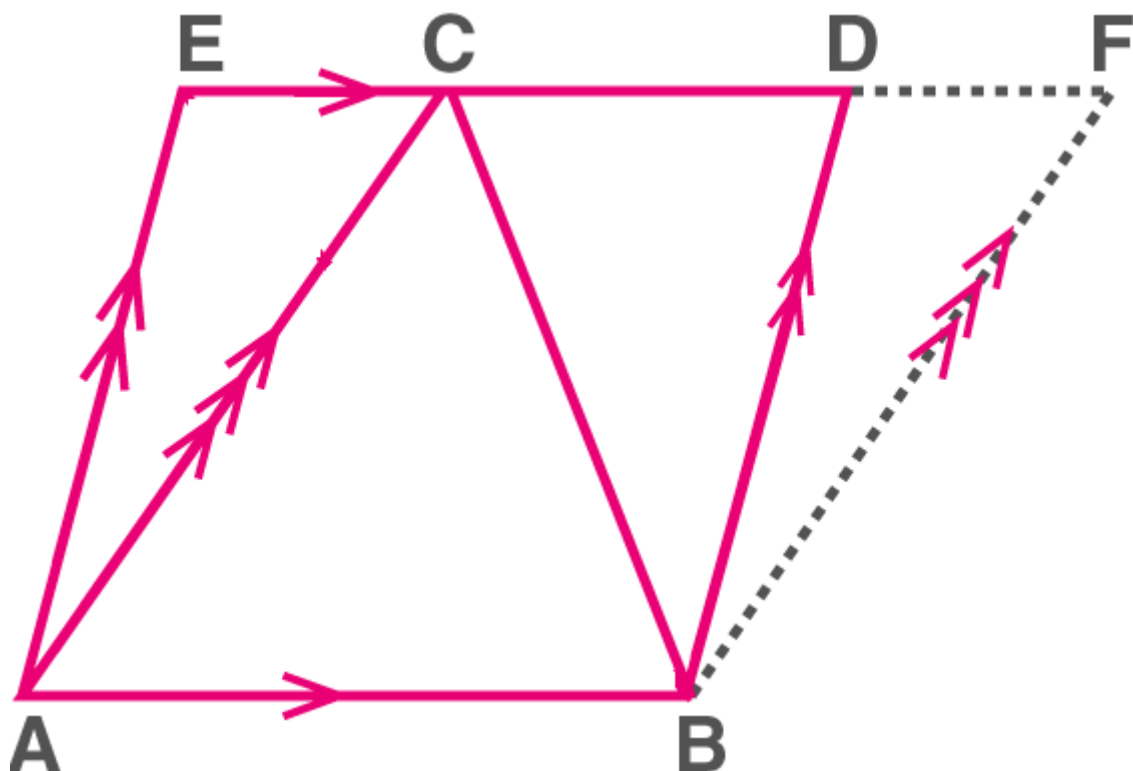


If two triangles have equal bases and are equal in area, then their corresponding altitudes are equal.

A Parallelogram and a Triangle Between the Same parallels

A triangle and a parallelogram are said to be on the same base and between the same parallels if

- They have a common side.
- The vertices opposite the common side lie on a straight line parallel to the common side.

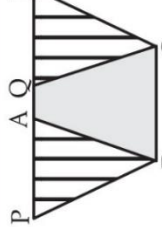
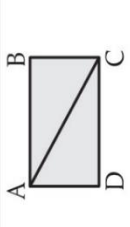
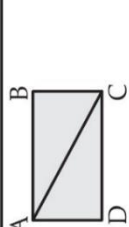


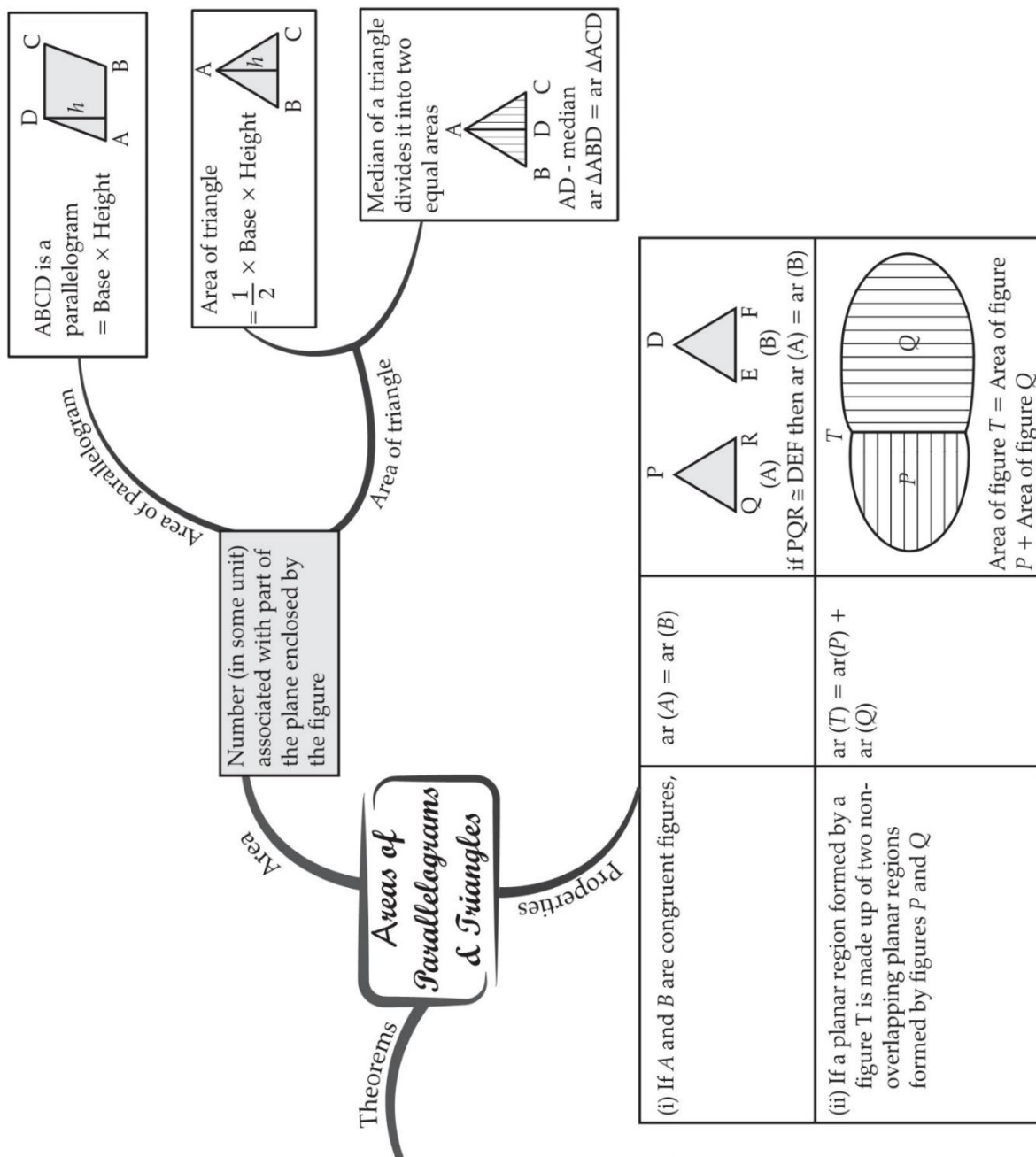
A triangle ABC and a parallelogram ABDE

Theorem: If a triangle and a parallelogram are on the common base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

Here, $\text{ar}(\triangle ABC) = \frac{1}{2} \text{ar}(\text{parallelogram ABDE})$

CHAPTER : 9 AREAS OF PARALLELOGRAMS & TRIANGLES

Statement	Figure
1.1. Parallelograms on the same base and between same parallels are equal in area.	 <p>$\text{ar}(\text{ABCD}) = \text{ar}(\text{PQCD})$</p>
2. Two triangles on the same base (or equal bases) and between the same parallels are equal in area	 <p>$\text{ar} \triangle ADC = \text{ar} \triangle ABC$</p>
3. Two triangles having the same base (or equal bases) and equal areas lie between the same parallels	 <p>if $\text{ar} \triangle ADC = \text{ar} \triangle ABC$ then $AB \parallel CD$</p>



Important Questions

Multiple Choice questions-

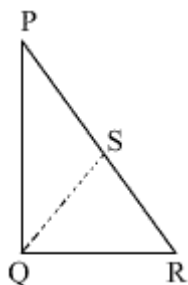
Question 1. What is the area of a parallelogram?

- (a) $\frac{1}{2} \times \text{Base} \times \text{Altitude}$
- (b) $\text{Base} \times \text{Altitude}$
- (c) $\frac{1}{4} \times \text{Base} \times \text{Median}$
- (d) $\text{Base} \times \text{Base}$

Question 2. AE is a median to side BC of triangle ABC. If $\text{area}(\triangle ABC) = 24\text{cm}$, then $\text{area}(\triangle ABE) =$

- (a) 8cm
- (b) 12cm
- (c) 16cm
- (d) 18cm

Question 3. In the figure, $\angle PQR = 90^\circ$, $PS = RS$, $QP = 12\text{cm}$ and $QS = 6.5\text{cm}$. The area of $\triangle PQR$ is



- (a) 30cm^2
- (b) 20cm^2
- (c) 39cm^2
- (d) 60cm^2

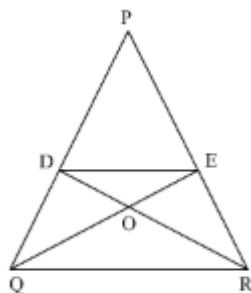
Question 4. ABCD is quadrilateral whose diagonal AC divides it into two parts, equal in area, then ABCD

ABCD is quadrilateral whose diagonal AC divides it into two parts, equal in area, then ABCD

- (a) Is a rectangles
- (b) Is a parallelogram
- (c) Is a rhombus

(d) Need not be any of (a), (b) or (c).

Question 5. In ΔPQR , if D and E are points on PQ and PR respectively such that $DE \parallel QR$, then ar (PQE) is equal to



(a) ar (PRD)

(b) ar (DQM)

(c) ar (PED)

(d) ar (DQR)

Question 6. If Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at O. Then,

(a) ar (AOD) = ar (BOC)

(b) ar (AOD) > ar (BOC)

(c) ar (AOD) < ar (BOC)

(d) None of the above

Question 7. For two figures to be on the same base and between the same parallels, one of the lines must be.

(a) Making an acute angle to the common base

(b) The line containing the common base

(c) Perpendicular to the common base

(d) Making an obtuse angle to the common base

Question 8. Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is:

(a) 1 : 3

(b) 1 : 2

(c) 2 : 1

(d) 1 : 1

Question 9. If P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD, then:

(a) ar (APB) > ar (BQC)

(b) $\text{ar}(\text{APB}) < \text{ar}(\text{BQC})$

(c) $\text{ar}(\text{APB}) = \text{ar}(\text{BQC})$

(d) None of the above

Question 10. A triangle and a rhombus are on the same base and between the same parallels. Then the ratio of area of triangle to that rhombus is:

(a) 1 : 3

(b) 1 : 2

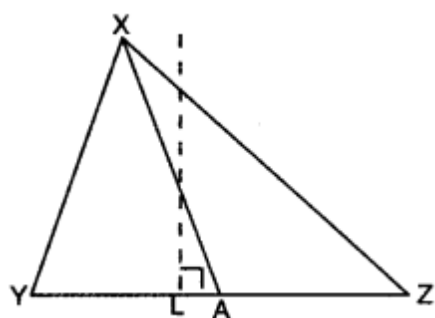
(c) 1 : 1

(d) 1 : 4

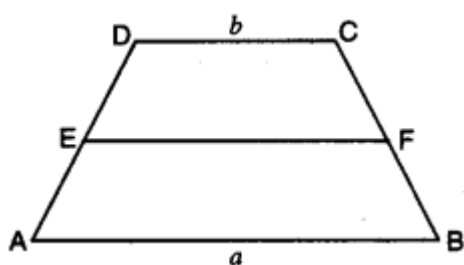
Very Short:

1. Two parallelograms are on equal bases and between the same parallels. Find the ratio of their areas.

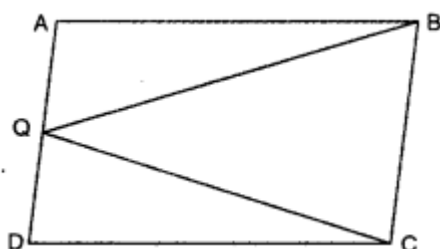
2. In $\triangle XYZ$, XA is a median on side YZ . Find ratio of $\text{ar}(\triangle XYA) : \text{ar}(\triangle XZA)$



3. ABCD is a trapezium with parallel sides $AB = a$ cm and $DC = b$ cm (fig.). E and F are the mid-points of the non parallel sides. Find the ratio of $\text{ar}(\text{ABFE})$ and $\text{ar}(\text{EFCD})$.

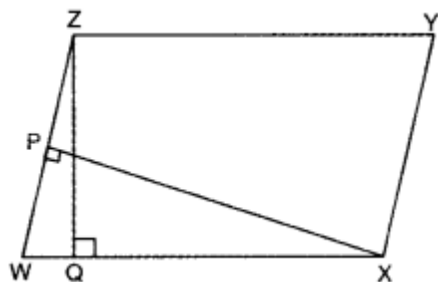


4. ABCD is a parallelogram and Q is any point on side AD. If $\text{ar}(\triangle QBC) = 10 \text{ cm}^2$, find $\text{ar}(\triangle QAB) + \text{ar}(\triangle QDC)$.

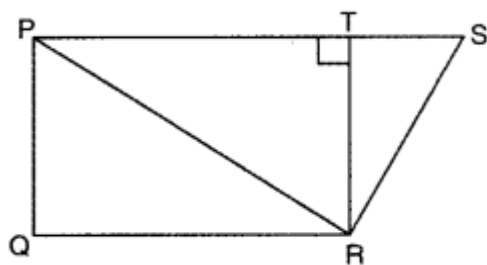


5. WXYZ is a parallelogram with $XP \perp WZ$ and $ZQ \perp WX$. If $WX = 8$ cm, $XP = 8$

cm and $ZQ = 2$ cm, find YX .

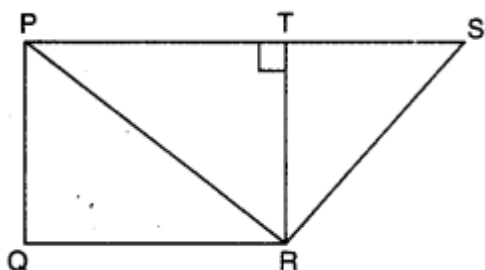


6. In figure, $TR \perp PS$, $PQ \parallel TR$ and $PS \parallel QR$. If $QR = 8$ cm, $PQ = 3$ cm and $SP = 12$ cm, find $\text{ar}(\text{quad. PQRS})$.



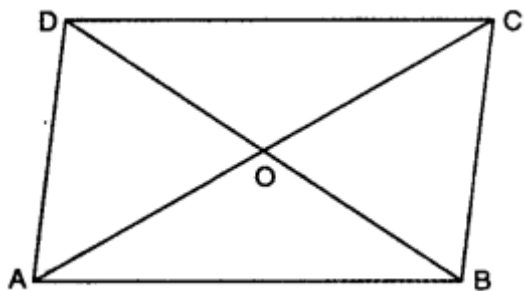
7. In the given figure, ABCD is a parallelogram and L is the mid-point of DC. If $\text{ar}(\text{quad. ABCL})$ is 72 cm, then find $\text{ar}(\triangle ADC)$.

8. In figure, $TR \perp PS$, $PQ \parallel TR$ and $PS \parallel QR$. If $QR = 8$ cm, $PQ = 3$ cm and $SP = 12$ cm, find $\text{ar}(\text{PQRS})$.

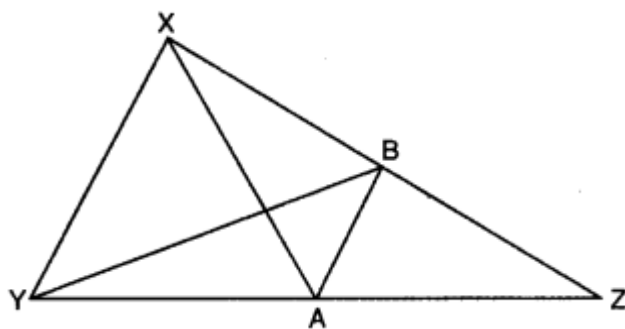


Short Questions:

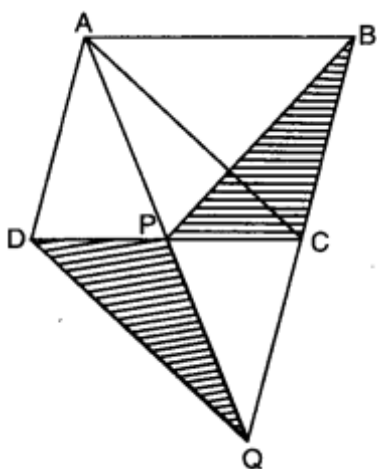
1. ABCD is a parallelogram and O is the point of intersection of its diagonals. If $\text{ar}(\triangle AOD) = 4 \text{ cm}^2$ find area of parallelogram ABCD.



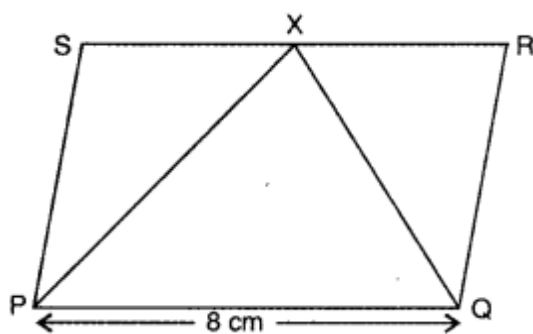
2. In the given figure of $\triangle XYZ$, XA is a median and $AB \parallel YX$. Show that YB is also a median.



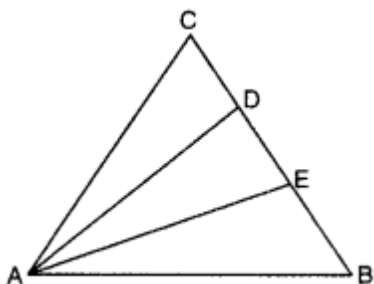
3. ABCD is a trapezium. Diagonals AC and BD intersect each other at O. Find the ratio $\text{ar}(\triangle AOD) : \text{ar}(\triangle BOC)$.
4. ABCD is a parallelogram and BC is produced to a point Q such that $AD = CQ$ (fig.). If AQ intersects DC at P, show that $\text{ar}(\triangle BPC) = \text{ar}(\triangle DPQ)$.



5. In the figure, PQRS is a parallelogram with $PQ = 8 \text{ cm}$ and $\text{ar}(\triangle PXQ) = 32 \text{ cm}^2$. Find the altitude of gm PQRS and hence its area.

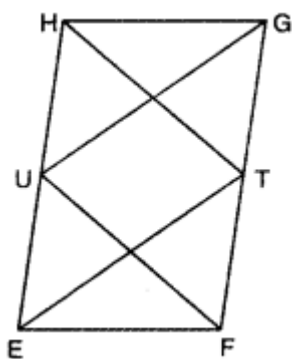


6. In $\triangle ABC$, D and E are points on side BC such that $CD = DE = EB$. If $\text{ar}(\triangle ABC) = 27 \text{ cm}^2$, find $\text{ar}(\triangle ADE)$

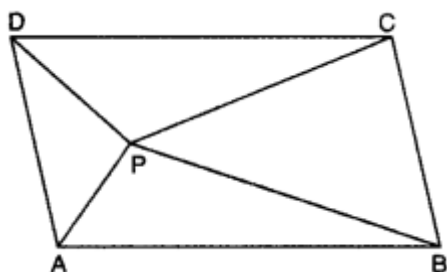


Long Questions:

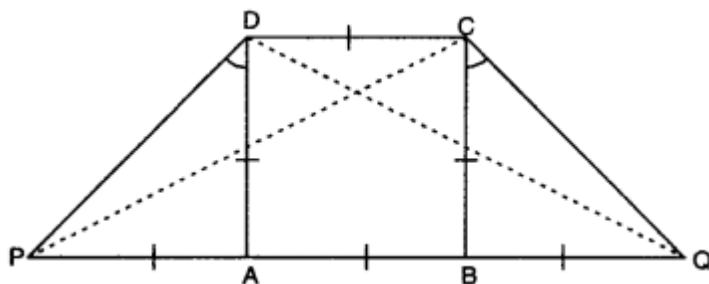
1. EFGH is a parallelogram and U and T are points on sides EH and GF respectively. If $\text{ar}(\triangle EHT) = 16\text{cm}$, find $\text{ar}(\triangle GUF)$.



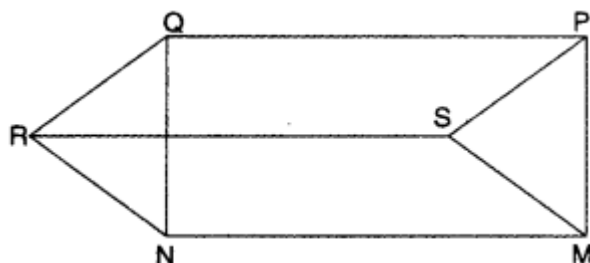
2. ABCD is a parallelogram and P is any point in its interior. Show that:
 $\text{ar}(\triangle APB) + \text{ar}(\triangle CPD) = \text{ar}(\triangle BPC) + \text{ar}(\triangle APD)$



3. In the given figure, ABCD is a square. Side AB is produced to points P and Q in such a way that $PA = AB = BQ$. Prove that $DQ = CP$.



4. In the given figure, PQRS, SRNM and PQNM are parallelograms, Show that :
 $\text{ar}(\triangle PSM) = \text{ar}(\triangle QRN)$.



5. Naveen was having a plot in the shape of a quadrilateral. He decided to donate some portion of it to construct a home for orphan girls. Further he decided to buy a land in lieu of his donated portion of his plot so as to form a triangle.
 (i) Explain how this proposal will be implemented?

(ii) Which mathematical concept is used in it?

(iii) What values are depicted by Naveen?

Assertion and Reason Questions-

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

- a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
- b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- c) Assertion is correct statement but reason is wrong statement.
- d) Assertion is wrong statement but reason is correct statement.

Assertion: The area of a parallelogram and a rectangle having a common base and between same parallels are equal.

Reason: Another name of a rectangle is a parallelogram.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

- a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
- b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
- c) Assertion is correct statement but reason is wrong statement.
- d) Assertion is wrong statement but reason is correct statement.

Assertion: The parallelogram on the same and between the same parallel are equal in area.

Reason: The areas of parallelogram between the same parallels are equal

Answer Key:

MCQ:

- 1. (b) Base \times Altitude
- 2. (b) 12cm
- 3. (c) 30cm^2
- 4. (d) Need not be any of (a), (b) or (c).
- 5. (a) ar (PRD)
- 6. (a) ar (AOD) = ar (BOC)
- 7. (b) The line containing the common base

8. (d) 1 : 1
 9. (c) $\text{ar}(\text{APB}) = \text{ar}(\text{BQC})$
 10.(b) 1 : 2

Very Short Answer:

1. 1:1 [\because Two parallelograms on the equal bases and between the same parallels are equal in

area.]

2. Here, XA is the median on side YZ.

$$\therefore \text{YA} = \text{AZ}$$

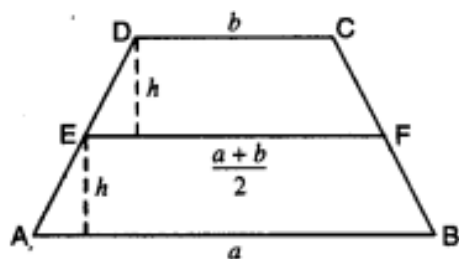
Draw $\text{XL} \perp \text{YZ}$

$$\therefore \text{ar}(\triangle \text{XYA}) = \frac{1}{2} \times \text{YA} \times \text{XL}$$

$$\text{ar}(\triangle \text{XZA}) = \frac{1}{2} \times \text{AZ} \times \text{XL}$$

$$\begin{aligned} \text{Thus, ar}(\triangle \text{XYA}) : \text{ar}(\triangle \text{XZA}) &= \frac{1}{2} \times \text{YA} \times \text{XL} : \frac{1}{2} \times \text{AZ} \times \text{XL} \\ &= 1 : 1 \quad [\because \text{YA} = \text{AZ}] \end{aligned}$$

- 3.



$$\text{Clearly, } \text{EF} = \frac{\text{AB} + \text{DC}}{2} = \frac{a+b}{2}$$

Let h be the height, then

$$\text{ar}(\text{Trap. ABFE}) : \text{ar}(\text{Trap. EFCD})$$

$$\Rightarrow \frac{1}{2} \left[a + \left(\frac{a+b}{2} \right) \right] \times h : \frac{1}{2} \left[b + \left(\frac{a+b}{2} \right) \right] \times h$$

$$\Rightarrow \frac{2a+a+b}{2} : \frac{2b+a+b}{2}$$

$$\Rightarrow 3a+b : 3b+a$$

4. Here, $\triangle \text{QBC}$ and parallelogram ABCD are on the same base BC and lie between the same parallels BC || AD.

$$\therefore \text{ar}(\text{||gm ABCD}) = 2 \text{ar}(\triangle \text{QBC}) \quad \text{ar}(\triangle \text{QAB}) + \text{ar}(\triangle \text{QDC}) + \text{ar}(\triangle \text{QBC}) = 2 \text{ar}(\triangle \text{QBC})$$

$$\text{ar}(\triangle QAB) + \text{ar}(\triangle QDC) = \text{ar}(\triangle QBC)$$

$$\text{Hence, ar}(\triangle QAB) + \text{ar}(\triangle QDC) = 10\text{cm}^2$$

$$[\because \text{ar}(\triangle QBC) = 10\text{ cm}^2 \text{ (given)}]$$

$$5. \text{ar}(\parallel\text{gm } WXYZ) = \text{ar}(\parallel\text{gm } WXYZ)$$

$$WX \times ZQ = WZ \times XP$$

$$8 \times 2 = WZ \times 8$$

$$\Rightarrow WZ = 2\text{cm}$$

$$\text{Now, } YX = WZ = 2\text{cm}$$

$$[\because \text{opposite sides of parallelogram are equal}]$$

6. Here,

$$PS \parallel QR \text{ [given]}$$

\therefore PQRS is a trapezium

$$\text{Now, } TR \perp PS \text{ and } PQ \parallel TR \text{ [given]}$$

$$\Rightarrow PQ \perp PS$$

$$\therefore PQ = TR = 3\text{cm [given]}$$

$$\text{Now, ar}(\text{quad. PQRS}) = \frac{1}{2} (PS + QR) \times PQ = \frac{1}{2} (12 + 8) \times 3 = 30\text{cm}^2$$

7. In $\parallel\text{gm } ABCD$, AC is the diagonal

$$\therefore \text{ar}(\triangle ABC) = \text{ar}(\triangle ADC) = \frac{1}{2} \text{ar}(\parallel\text{gm } ABCD)$$

In $\triangle ADC$, AL is the median

$$\therefore \text{ar}(\triangle ADL) = \text{ar}(\triangle ACL) = \frac{1}{2} \text{ar}(\triangle ADC) = \frac{1}{4} \text{ar}(\parallel\text{gm } ABCD)$$

$$\text{Now, ar}(\text{quad. } ABCL) = \text{ar}(\triangle ABC) + \text{ar}(\triangle ACL) = \frac{3}{4} \text{ar}(\parallel\text{gm } ABCD)$$

$$\Rightarrow 72 \times \frac{4}{3} = \text{ar}(\parallel\text{gm } ABCD)$$

$$\Rightarrow \text{ar}(\parallel\text{gm } ABCD) = 96\text{ cm}^2$$

$$\therefore \text{ar}(\triangle ADC) = \frac{1}{2} \text{ar}(\parallel\text{gm } ABCD) = \frac{1}{2} \times 96 = 48\text{ cm}^2$$

8. Here, $PS \parallel QR$

\therefore PQRS is a trapezium in which $PQ = 3\text{cm}$, $QR = 8\text{cm}$ and $SP = 12\text{cm}$

$$\text{Now, } TR \perp PS \text{ and } PQ \parallel TR$$

\therefore PQRT is a rectangle

$$[\because PQ \parallel TR, PT \parallel QR \text{ and } \angle PTR = 90^\circ]$$

$$\Rightarrow PQ = TR = 3\text{cm}$$

$$\text{Now, ar(PQRS)} = \frac{1}{2} (PS + QR) \times TR = \frac{1}{2} (12 + 8) \times 3 = 30\text{cm}^2.$$

Short Answer:

Ans: 1. Here, ABCD is a parallelogram in which its diagonals AC and BD intersect each other in O.

\therefore O is the mid-point of AC as well as BD.

Now, in $\triangle ADB$, AO is its median

$$\therefore \text{ar}(\triangle ADB) = 2 \text{ar}(\triangle AOD)$$

[\because median divides a triangle into two triangles of equal areas]

$$\text{So, ar}(\triangle ADB) = 2 \times 4 = 8\text{cm}^2$$

Now, $\triangle ADB$ and \square ABCD lie on the same base AB and lie between same parallels AB and CD

$$\therefore \text{ar}(ABCD) = 2 \text{ar}(\triangle ADB).$$

$$= 2 \times 8$$

$$= 16\text{cm}^2$$

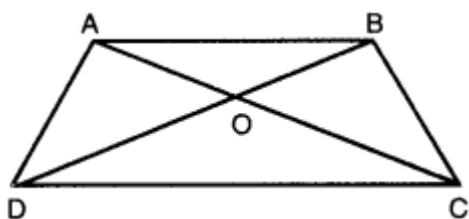
Ans: 2. Here, in $\triangle XYZ$, AB \parallel YX and XA is a median.

\therefore A is the mid-point of YZ. Now, AB is a line segment from mid-point of one side (YZ) and parallel to another side (AB \parallel YX), therefore, it bisects the third side XZ.

\Rightarrow B is the mid-point of XZ.

Hence, YB is also a median of $\triangle XYZ$.

Ans: 3.



Here, ABCD is a trapezium in which diagonals AC and BD intersect each other at O.

$\triangle ADC$ and $\triangle BCD$ are on the same base DC and between the same 'parallels i.e., AB \parallel DC.

$$\therefore \text{ar}(\triangle ADC) = \text{ar}(\triangle BCD)$$

$$\Rightarrow \text{ar}(\triangle AOD) + \text{ar}(\triangle ODC)$$

$$= \text{ar}(\triangle AOB) + \text{ar}(\triangle ODC)$$

$$\Rightarrow \text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$$

$$\Rightarrow \frac{\text{ar}(\triangle AOD)}{\text{ar}(\triangle BOC)} = 1$$

Ans: 4. In \square ABCD,

$$\text{ar}(\triangle APC) = \text{ar}(\triangle BCP) \dots(i)$$

[\because triangles on the same base and between the same parallels have equal area]

$$\text{Similarly, } \text{ar}(\triangle ADQ) = \text{ar}(\triangle ADC) \dots(ii)$$

$$\text{Now, } \text{ar}(\triangle ADQ) - \text{ar}(\triangle ADP) = \text{ar}(\triangle ADC) - \text{ar}(\triangle ADP)$$

$$\text{ar}(\triangle DPQ) = \text{ar}(\triangle ACP) \dots (iii)$$

From (i) and (iii), we have

$$\text{ar}(\triangle BCP) = \text{ar}(\triangle DPQ)$$

$$\text{or } \text{ar}(\triangle BPC) = \text{ar}(\triangle DPQ)$$

Ans: 5. Since parallelogram PQRS and APXQ are on the same base PQ and lie between the same

parallels PQ || SR

\therefore Altitude of the $\triangle PXQ$ and ||gm PQRS is same.

$$\text{Now, } \frac{1}{2} \text{ PQ} \times \text{altitude} = \text{ar}(\triangle PXQ)$$

$$\Rightarrow \frac{1}{2} \times 8 \times \text{altitude} = 32$$

$$\text{altitude} = 8\text{cm}$$

$$\text{ar}(\text{||gm PQRS}) = 2 \text{ ar}(\triangle PXQ)$$

$$= 2 \times 32 = 64\text{cm}^2$$

Hence, the altitude of parallelogram PQRS is 8cm and its area is 64cm^2 .

Ans: 6. Since in $\triangle AEC$, $CD = DE$, AD is a median.

$$\therefore \text{ar}(\triangle ACD) = \text{ar}(\triangle ADE)$$

[\because median divides a triangle into two triangles of equal areas]

Now, in $\triangle ABD$, $DE = EB$, AE is a median

$$\text{ar}(\triangle ADE) = \text{ar}(\triangle AEB) \dots (ii)$$

From (i), (ii), we obtain

$$\text{ar}(\triangle ACD) = \text{ar}(\triangle ADE) = \text{ar}(\triangle AEB) \frac{1}{3} \text{ar}(\triangle ABC)$$

$$\therefore \text{ar}(\triangle ADE) = \frac{1}{3} \times 27 = 9\text{cm}^2$$

Long Answer:

$$\text{Ans: 1. } \therefore \text{ar}(\triangle EHT) = \frac{1}{2} \text{ar}(\text{||gm EFGH}) \dots(i)$$

Similarly, $\triangle GUF$ and parallelogram EFGH are on the same base GF and lie between the same parallels GF and HE

$$\therefore \text{ar}(\triangle GUF) = \frac{1}{2} \text{ar}(\text{||gm EFGH}) \dots(ii)$$

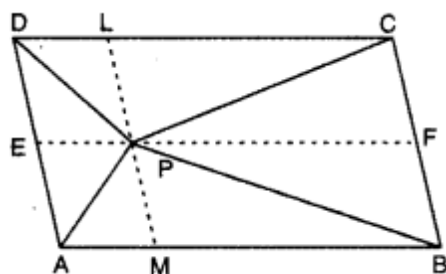
From (i) and (ii), we have

$$\begin{aligned} \text{ar}(\Delta GUF) &= \text{ar}(\Delta EHT) \\ &= 16\text{cm}^2 [\because \text{ar}(\Delta EHT) = 16\text{cm}^2] \text{ [given]} \end{aligned}$$

Ans: 2. Through P, draw a line LM || DA and EF || AB

Since ΔAPB and || gm ABFE are on the same base AB and lie between the same parallels AB and EF.

$$\therefore \text{ar}(\Delta APB) = \frac{1}{2} \text{ar}(\text{|| gm ABFE}) \dots (i)$$



Similarly, ΔCPD and parallelogram DCFE are on the same base DC and between the same parallels DC and EF.

$$\therefore \text{ar}(\Delta CPD) = \frac{1}{2} \text{ar}(\text{|| gm DCFE}) \dots (ii)$$

Adding (i) and (ii), we have

$$\begin{aligned} \text{ar}(\Delta APB) + \text{ar}(\Delta CPD) &= \frac{1}{2} \text{ar}(\text{|| gm ABFE}) + \text{ar}(\text{|| gm DCFE}) \\ &= \frac{1}{2} \text{ar}(\text{|| gm ABCD}) \dots (iii) \end{aligned}$$

Since ΔAPD and parallelogram ADLM are on the same base AD and between the same parallels AD and ML

$$\therefore \text{ar}(\Delta APD) = \frac{1}{2} \text{ar}(\text{|| gm ADLM}) \dots (iv)$$

$$\text{Similarly, } \text{ar}(\Delta BPC) = \frac{1}{2} \text{ar}(\text{|| gm BCLM}) \dots (v)$$

Adding (iv) and (v), we have

$$\text{ar}(\Delta APD) + \text{ar}(\Delta BPC) = \frac{1}{2} \text{ar}(\text{|| gm ABCD}) \dots (vi)$$

From (iii) and (vi), we obtain

$$\text{ar}(\Delta APB) + \text{ar}(\Delta CPD) = \text{ar}(\Delta APD) + \text{ar}(\Delta BPC)$$

Ans: 3. In ΔPAD , $\angle A = 90^\circ$ and $DA = PA = AB$

$$\Rightarrow \angle ADP = \angle APD = \frac{90^\circ}{2} = 45^\circ$$

Similarly, in ΔQBC , $\angle B = 90^\circ$ and $BQ = BC = AB$

$$\Rightarrow \angle BCQ = \angle BQC = \frac{90^\circ}{2} = 45^\circ$$

In ΔPAD and ΔQBC , we have

$$PA = BQ \text{ [given]}$$

$$\angle A = \angle B \text{ [each} = 90^\circ]$$

$$AD = BC \text{ [sides of a square]}$$

$$\Rightarrow \triangle PAD \cong \triangle QBC \text{ [by SAS congruence rule]}$$

$$\Rightarrow PD = QC \text{ [c.p.c.t.]}$$

Now, in $\triangle PDC$ and $\triangle QCD$

$$DC = DC \text{ [common]}$$

$$PD = QC \text{ [prove above]}$$

$$\angle PDC = \angle QCD \text{ [each} = 90^\circ + 45^\circ = 135^\circ]$$

$$\Rightarrow \triangle PDC \cong \triangle QCD \text{ [by SAS congruence rule]}$$

$$\Rightarrow PC = QD \text{ or } DQ = CP$$

Ans: 4. Since PQRS is a parallelogram.

$$\therefore PS = QR \text{ and } PS \parallel QR$$

Since SRNM is also a parallelogram.

$$\therefore SM = RN \text{ and } SM \parallel RN$$

Also, PQNM is a parallelogram

$$\therefore PM \parallel QN \text{ and } PM = QN$$

Now, in $\triangle PSM$ and $\triangle QRN$

$$PS = QR$$

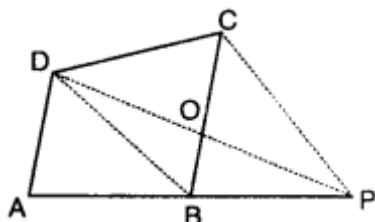
$$SM = RN$$

$$PM = QN$$

$$\triangle PSM \cong \triangle QRN \text{ [by SSS congruence axiom]}$$

$$\therefore \text{ar}(\triangle PSM) = \text{ar}(\triangle QRN) \text{ [congruent triangles have same areas]}$$

Ans: 5.



- (i) Let ABCD be the plot and Naveen decided to donate some portion to construct a home for orphan girls from one corner say C of plot ABCD. Now, Naveen also purchases equal amount of land in lieu of land CDO, so that he may have triangular form of plot. BD is joined. Draw a line through C parallel to DB to meet AB produced in P.

Join DP to intersect BC at O.

Now, ABCD and ABPD are on the same base and between same parallels CP || DB.

$$\text{ar}(\Delta BCD) = \text{ar}(\Delta BPD) \quad \text{ar}(\Delta COD) + \text{ar}(\Delta DBO) = \text{ar}(\Delta BOP) + \text{ar}(\Delta DBO)$$

$$\text{ar}(\Delta COD) = \text{ar}(\Delta BOP) \quad \text{ar}(\text{quad. ABCD})$$

$$= \text{ar}(\text{quad. ABOD}) + \text{ar}(\Delta COD)$$

$$= \text{ar}(\text{quad. ABOD}) + \text{ar}(\Delta BOP)$$

$$[\because \text{ar}(\Delta COD) = \text{ar}(\Delta BOP)] \text{ (proved above)}$$

$$= \text{ar}(\Delta APD)$$

Hence, Naveen purchased the portion ABOP to meet his requirement.

(ii) Two triangles on the same base and between same parallels are equal in area.

(iii) We should help the orphan children.

Assertion and Reason Answers-

1. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. c) Assertion is correct statement but reason is wrong statement.